Taking formally the limit $N \to \infty$ in (3), we have

$$\lim_{N\to\infty} T(t)/T_0 = 2\pi^{-1}\int_0^\infty dx\, x^{-1} J_0(tx)\sin x \ ,$$

equal to[2] the right-hand side of Eq. (2). The formal step involved is easily justified but details are omitted here.

[1]R. J. Rubin, Phys. Rev. 131, 964 (1963). See in particular his Appendix C.

[2]W. Magnus and F. Oberhettinger, *Formulas and Theorems for the Special Functions of Mathematical Physics* (Chelsea Publishing Co., New York, 1949), pp. 32–34.

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Specular Reflection in Al Films

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Experimental results on the size-affected resistivity of aluminum films are discussed.

We wish to comment on two conclusions reached by von Bassewitz and Mitchell[1] (BM) from their experimental results:

(a) The value of the mean free path l, for which they give a value of 17.5 μ for polycrystal films as well as for single-crystal films.

(b) The reflection coefficient p, BM claim that the reflection at a smooth surface of a single-crystal film of Al is nearly specular.

1. DETERMINATION OF l

To obtain the mean free path l from the measurements of the size-affected resistivity ρ, BM plot $1/\rho t$ (t is the thickness of the film) as a function of $\ln t$ and draw a straight line through the experimental points. Extrapolation of the line of the axis where $1/\rho t = 0$ gives an intersection point $t_1 = 17.5\ \mu$. BM conclude that the intersection point should be the point where $t_1/l = 1$. This leads to $l = 17.5\ \mu$ and $\rho_\sim l = 11.5\times10^{-12}\,\Omega\ \text{cm}^2$, where $\rho_\sim$ is the bulk resistivity. To obtain this, BM use an approximation of the Fuchs-Sondheimer[2] (FS) theory for the case $t/l \ll 1$ which reads

$$\rho_\sim/\rho = (3t/4l)(1+p)/(1-p)\ \ln(l/t) \quad . \qquad (1)$$

If $l = 17.5\ \mu$, then the experiments of BM were done in a regime $0.01 < t/1 < 0.2$. However, in this regime the extrapolation procedure described above is misleading. This is most easily seen by plotting $1/\rho_{\text{th}} t$ versus $\ln t$, where ρ_{th} is the (numerical) value of the full FS theory. When a straight line is drawn through the calculated points in the regime $0.01 \leqslant t/l \leqslant 0.2$, one finds that extrapolation to $1/\rho_{\text{th}} t = 0$ gives $t_i/l = 1.9$ if $p = 0$, $t_i/l = 1.3$ if $p = 0.1$, $t_i/l = 0.9$ if $p = 0.3$, and $t_i/l = 0.75$ if $p = 0.5$, instead of $t_i/l = 1$ as follows from Eq. (1). These differences clearly show that if $l = 17.5\ \mu$, Eq. (1) may not be used. The procedure used by BM to obtain $l = 17.5\ \mu$ and thus $\rho_\sim l = 11.5\times10^{-12}\,\Omega\,\text{cm}^2$ is, in fact, inconsistent with the FS theory.

2. DISCUSSION OF REFLECTION COEFFICIENT p

When concluding something about the reflection coefficient p from the FS theory, the dependence of l on t must be known. As BM have done no other experiments from which l can be determined, they had to make an assumption about the dependence of l on t. In fact, they reason in terms of a constant mean free path and then conclude from the fact that the slope of the straight line in the $1/\rho t$-versus-$\ln t$ graph is different for single-crystal films and polycrystal films that $p \neq 0$. Using Eq. (1) they found that for smooth surfaces of single-crystal films $0.42 < p < 0.84$. From the discussion given above about the validity of Eq. (1) for the experiments of BM, it is clear that the obtained value of p seems not to be correct. The conclusion that $p \neq 0$ can, however, be right if indeed l is independent of t. It is clear, however, that one could also interpret the experiments by assuming $p = 0$ and letting l be a function of t, a possibility which is not at all unlikely in view of the results obtained by Cotti *et al.*[3] Let us consider the consequences of such an interpretation; BM give that ρ varies from 8.5×10^{-8} to 1.1×10^{-8} Ω cm for single-crystal films, and from 18×10^{-8} to 2.0×10^{-8} Ω cm for polycrystal films for thicknesses between 0.2 and 3.6 μ. Using the results of a two-band model,[4] one finds that for single-crystal films, l varies from 6.1 to 18 μ and from

1.2 to 5.5 μ for polycrystal films for thicknesses between 0.2 and 3.6 μ. We thus obtain the reasonable result that if diffuse scattering is assumed, l is larger for single-crystal films than for polycrystal films, and further, that l decreases with decreasing t in such a way that t/l also decreases so that the size effects become stronger. This latter result is similar to the results of Cotti *et al.*,[3] who showed for In and Al assuming diffuse scattering for polycrystal sheets, that l decreases with decreasing t. This does not mean that exactly the same dependence of l on t should occur for the films used by BM as their preparation technique is different from that of Cotti. As stated, Cotti treated $p=0$, so it seems worthwhile to consider the consequences for l if the experiments of Cotti are interpreted with $p \neq 0$. Cotti[3] determined the dc resistivity ρ and the decay time τ of eddy currents, which quantity can be expressed in terms of a resistivity ρ_τ. If p is used as a parameter, the value of $l(p)$ can be directly determined by comparing the experimental value of ρ_τ/ρ with the results of the calculations of FS and of Brändli and Cotti.[5] The mean free path obtained in this way is thus the bulk mean free path as a function of the assumed reflection coefficient p. In Fig. 1 we have plotted the values of l as a function of p for some specimens used by Cotti. It is clear that independent of the assumption made about p, the experimental results show a marked dependence of l on t. In view of this evidence, we believe that a dependence of l on t may also be present in the experiments of BM. In our opinion this means that the experiments of BM give no conclusive evidence for specular reflection in Al.

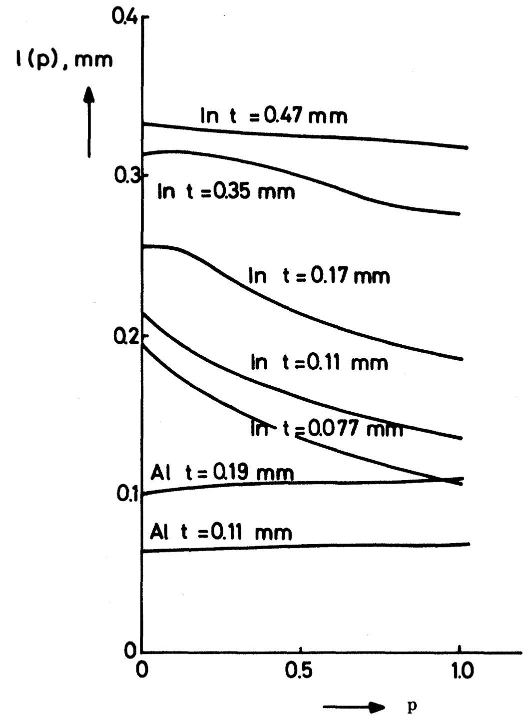

FIG. 1. Mean free path $l(p)$ as a function of the assumed reflection coefficient p for some indium and aluminum specimens as used by Cotti *et al.* (Ref. 3).

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[4]W. F. Druyvesteyn, Phil. Mag. 18, 11 (1968).

[5]G. Brändli and P. Cotti, Helv. Phys. Acta 38, 801 (1965).